

24018

B. Tech Common for all branches 2nd Semester**F. Scheme Examination,****May-2015****MATHEMATICS-II****Paper-MATH-102-F***Time allowed : 3 hours]**[Maximum marks : 100*

Note : Question No. 1 is compulsory. Attempt total five questions with selecting one question from each Unit. All questions carry equal marks.

1. (a) State Stoke's theorem. $2\frac{1}{2} \times 8$
- (b) Write inverse L. T. of $\frac{s}{(s^2 + a^2)^2}$
- (c) Solve $(x^2 - ay) dx - (ax - y^2) dy = 0$
- (d) Find unit vector normal to the surface $\phi = x^2 yz + xy^3 - 6xyz$ at the point $(1, 2, -1)$
- (e) Find the Orthogonal trajectories of $y^2 = 4ax$.
- (f) Find the P. I. of $(D^3 + 1)y = \sin(2x + 3)$.
- (g) Find the Laplace transform of $\frac{e^{-t} \sin t}{t}$.
- (h) Solve $z = px + qy + \sqrt{pq}$

24018-P-4-Q-9 (15)

[P.T.O.]

Unit-1

2. (a) Find the directional derivative of $\nabla \cdot (\nabla f)$ at the point $(1, -2, 1)$ in the direction of the normal to the surface $xy^2 z = 3x + z^2$, where $f = 2x^3 y^2 z^4$

- (b) (i) Prove that

$$\nabla \times (F \times G) = (\nabla \cdot G) F - (\nabla \cdot F) G + (G \cdot \nabla) F - (F \cdot \nabla) G$$

- (ii) If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, and

$r \neq 0$ show that

$$\nabla^2 (r^n) = n(n+1)r^{n-2}$$

3. (a) Verify Green's theorem in the plane for $\int_C [2y^2 dx + 3x dy]$,

where C is the boundary of the closed region bounded between $y = x$ and $y = x^2$.

- (b) Verify divergence theorem for $\vec{F} = 4x\hat{i} - 2y^2\hat{j} + z^2\hat{k}$ over the region bounded by the cylinder $x^2 + y^2 = 4, z = 0, z = 3$.

Unit-2

4. (a) Solve the following differential equation :

$$(xy^2 + 2x^2y^3) dx - (x^3y^2 - x^2y) dy = 0$$

- (b) When a resistance of R ohms is connected in series with an inductance of L henries with an emf of E volts, the current i amperes at time t is given

by $L \frac{di}{dt} + Ri = E$.

If $E = 10 \sin t$ volts and $i = 0$ when $t = 0$, find i as a function of t .

5. (a) Solve :

$$(3x+2)^2 \frac{d^2y}{dx^2} + 3(3x+2) \frac{dy}{dx} - 36y = 3x^2 + 4x + 1.$$

- (b) An electric circuit consists of an inductance of 0.1 Henry, a resistance of 20Ω and a condenser of capacitance 25 micro farads. Find the charge q and the current i at any given time t , given that at $t = 0$, $q = 0.05$ c, $i = \frac{dq}{dt} = 0$, when $t = 0$.

Unit-3

6. (a) Find the Laplace transform of
- $\sinh at \cos at$
 - Evaluate $\int_0^\infty t \sin 3t dt$.
- (b) Apply convolution theorem to evaluate

$$\mathcal{L}^{-1} \left\{ \frac{1}{(s^2 + 1)(s^2 + 9)} \right\}$$

7. (a) (i) Find Laplace transform of $f(t)$, where

$$f(t) = |t-1| + |t+1|, t > 0.$$

- (ii) Find Laplace Inverse of $\log \left(\frac{s+a}{s+b} \right)$

(b) Solve $\frac{d^2x}{dt^2} + 2 \frac{dx}{dt} + 5x = e^{-t} \sin t$

given that $x(0) = 0$ and $x'(0) = 1$ by using Laplace transform.

Unit-4

8. (a) Solve the following differential equation

$$x^2(y-z)p + y^2(z-x)q = z^2(x-y)$$

- (b) Solve the equation by Charpit's method

$$2z + p^2 + qy + 2y^2 = 0.$$

9. (a) Form partial differential equation from the relation

$$z = y^2 + 2f\left(\frac{1}{x} + \log y\right).$$

- (b) Using method of separation of variables,

$$4(\partial u / \partial x) + (\partial u / \partial y) = 3u, \text{ given}$$

$$u = 3e^{-y} - e^{-5y} \text{ when } x = 0$$